

# KINEMATICS

## Distance & Displacement

### DISTANCE

- \* Actual length of path.
- \* Scalar
- \* Depend on path
- \* always ↑ w.r.t time.
- \* Always ⊕ve.
- \* In close path not equal to zero.

### Displacement

\* Shortest dist. b/w Initial & Final point.

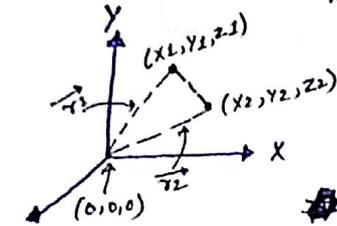
\* Disp vector

$$\vec{r} + \vec{r} = \vec{r}$$

$$\vec{r} = \vec{r}_2 = \vec{r}_1$$

$$\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

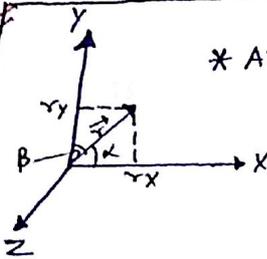


\* Angle from 'x'-axis

$$\cos \alpha = \frac{r_x}{|\vec{r}|} \Rightarrow \alpha = \cos^{-1} \left( \frac{r_x}{\sqrt{r_x^2 + r_y^2 + r_z^2}} \right)$$

\* Angle from y-axis

$$\cos \beta = \frac{r_y}{|\vec{r}|} = \beta = \cos^{-1} \left( \frac{r_y}{\sqrt{r_x^2 + r_y^2 + r_z^2}} \right)$$

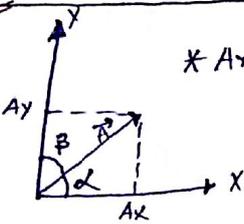


\* Angle from 'x'-axis

$$\tan \alpha = \frac{A_y}{A_x} = \alpha = \tan^{-1} \left( \frac{A_y}{A_x} \right)$$

\* Angle from 'y'-axis

$$\tan \beta = \frac{A_x}{A_y} = \beta = \tan^{-1} \left( \frac{A_x}{A_y} \right)$$



**NOTE** → It is vector quantity & depend on final & initial position. (Independent from path)  
 \* When particle goes away from initial point its value ↑ & move towards initial its value ↓. [Int may (↑) or (↓) w.r.t time].

\* Distance ≥ Displacement.

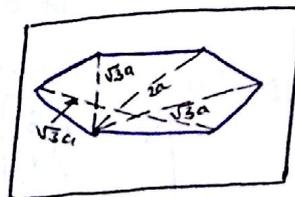
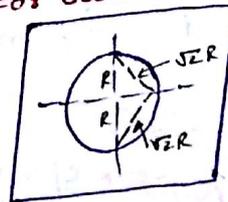
\* If particle move in a rectilinear path in same direction distance is equal to displacement & when direction change distance is always greater than displacement.

\* In close path disp. of moving particle is zero.

\*\*\*

# A particle move in circular path radius 'r'. then for calculation of distance & Displacement of particle.

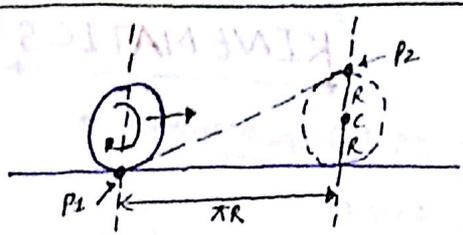
	Distance	Displacement
1) → $\frac{1}{4}$ th	$\frac{\pi}{2} R$	$\sqrt{2} R$
2) → $\frac{1}{2}$	$\pi R$	$2R$
3) → $\frac{3}{4}$	$\frac{3\pi}{2} R$	$\sqrt{2} R$
4) → 1	$2\pi R$	0
5) → In a 'θ' angular disp.	OR	$2R \sin(\theta/2)$



# iii →

$$P_1 P_2 = \sqrt{(\pi R)^2 + (2R)^2}$$

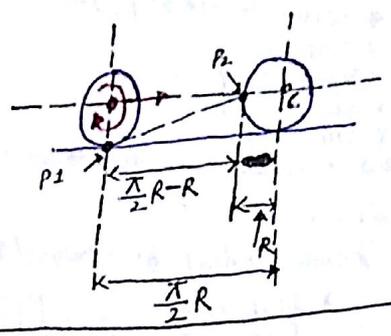
$$= R \sqrt{\pi^2 + 4}$$



\* iii →

$$P_1 P_2 = \sqrt{\left(\frac{\pi}{2}R - R\right)^2 + R^2}$$

$$= R \sqrt{\left(\frac{\pi}{2} - 1\right)^2 + 1}$$



# In circle  
In 'θ' Angular displacement

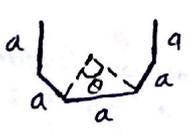
Disp =  $2R \sin(\theta/2)$

Distance =  $\theta R$

\* Average velocity in circle =  $\frac{v \sin(\theta/2)}{\theta/2}$

Change in velocity =  $2v \sin(\theta/2)$   
 faverage =  $\frac{mv^2 \sin(\theta/2)}{R \theta/2}$

# Polygon of side 'n'



\* Average velocity  
 $\frac{v \sin(m\theta/2)}{m \sin(\theta/2)}$

\* Then After side 'm'  
 Distance =  $ma$

Displacement =  $\frac{a \sin(m\theta/2)}{\sin(\theta/2)}$

|2| → Speed & velocity

Speed =  $\frac{\text{Distance}}{\text{Time}}$   
 ↳ scalar

velocity =  $\frac{\text{Displacement}}{\text{Time}}$   
 ↳ vector.

NOTE → Speed of moving particle is always ⊕ve but velocity may be ⊕ve, 0, or ⊖ve.

# Instantaneous speed/velo

$v = \frac{\Delta s}{\Delta t}$

$v_t = \lim_{\Delta t \rightarrow 0} \left| \frac{ds}{dt} \right|$

$\vec{v}_t = \lim_{\Delta t \rightarrow 0} \frac{d\vec{s}}{dt}$

⇒ slope of 's-t' curve.

## # Average speed/velocity

$$\vec{V}_{\text{Avg}} = \text{Avg. velo} = \frac{\text{Total disp.}}{\text{time}} = \frac{\vec{s}_1 + \vec{s}_2 + \dots + \vec{s}_N}{t_1 + t_2 + \dots + t_N}$$

$$\vec{V}_{\text{avg}} = \frac{\text{Total distance}}{\text{time}} = \frac{|\vec{s}_1| + |\vec{s}_2| + \dots + |\vec{s}_N|}{t_1 + t_2 + \dots + t_N}$$

NOTE → If particle move in a rectilinear path in same direction Avg. speed is equal to Avg. velo. & When direction change Avg. speed is always greater than Avg. velocity.

### Special case

Case-I → Particle covered  $x_1, x_2, x_3, \dots, x_N$  with speed  $v_1, v_2, v_3, \dots, v_N$ . Then Avg. speed of particle.

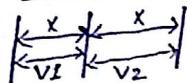
$$V_{\text{Avg}} = \frac{x_1 + x_2 + \dots + x_N}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \dots + \frac{x_N}{v_N}}$$

#  $x_1 = x_2 = \dots = x_N = x$

$$V_{\text{Avg}} = \frac{N}{\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_N}}$$

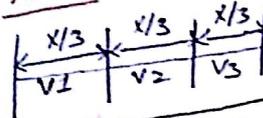
\*\*x

|a| →  $N=2$



$$* V_{\text{Avg}} = \frac{2v_1v_2}{v_1+v_2} \text{ (H.M)}$$

|b| →  $N=3$



$$* V_{\text{Avg}} = \frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$$

Case-II → Particle move with speed  $v_1, v_2, v_3, \dots, v_N$  for time interval  $t_1, t_2, t_3, \dots, t_N$  than Avg. speed of particle.

$$V_{\text{Avg}} = \frac{x_1 + x_2 + \dots + x_N}{t_1 + t_2 + \dots + t_N}$$

$$* V_{\text{Avg}} = \frac{v_1t_1 + v_2t_2 + \dots + v_Nt_N}{t_1 + t_2 + \dots + t_N}$$

#  $t_1 = t_2 = \dots = t_N = t$

$$V_{\text{Avg}} = \frac{v_1 + v_2 + \dots + v_N}{N}$$

|a| →  $N=2$

$$* V_{\text{Avg}} = \frac{v_1 + v_2}{2}$$

|b| →  $N=3$

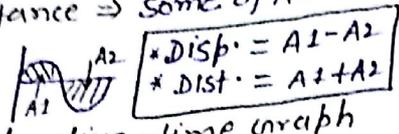
$$* V_{\text{Avg}} = \frac{v_1 + v_2 + v_3}{3}$$



→ Area enclosed b/w velocity-time curve & the time axis represent disp. of particle.

$$\text{Area} = \text{Integration} = \int y dx = \int v dt = ds = \text{displacement}$$

- \* Disp. = some of Area of v-t graph with proper sign.
- \* Dist. = Some of Area of v-t graph with proper sign.
- \* Distance ⇒ some of Area of v-t graph with +ve sign.



$$\begin{aligned} \text{* Disp.} &= A_1 - A_2 \\ \text{* Dist.} &= A_1 + A_2 \end{aligned}$$

\* Acceleration-time graph  
→ Acceleration-time-graph represent Jerk of particle.

$$\text{slope} = \frac{dv}{dx} = \frac{da}{dt} = \frac{a_2 - a_1}{t_2 - t_1} = \text{Jano} = \text{Jerk}$$

\* Area enclosed b/w acceleration-time curve & time axis represent change in velocity of particle of particle.

$$\text{Area} = I = \int y dx = \int a dt = dv = v_f - v_i$$

Imp  
\* If a particle start from with const. Accn. 'α' & comes in rest with constant retardation 'β' then Relation →

$$\frac{d_1}{d_2} = \frac{t_1}{t_2} = \frac{\beta}{\alpha}$$

NOTE → Direction of instantaneous Acceleration in the direction of force. but Avg. Acceleration in the direction of change in velocity.

\* Avg Accn.

$$\vec{A}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

Problem based on differentiation & integration

case-I  $x = f(t)$

$$\begin{aligned} v &= \frac{dx}{dt} = \frac{d}{dt} [f(t)] \\ a &= \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2} [f(t)] \end{aligned}$$

case-II

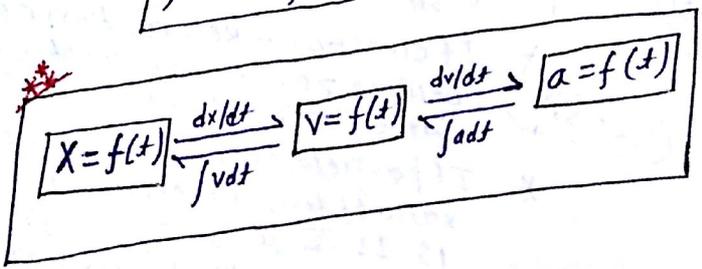
$$\begin{aligned} a &= f(t) \\ a &= \frac{dv}{dt} \\ dv &= \int a dt \\ v &= \frac{ds}{dt} \quad ds = \int v dt \end{aligned}$$

case-III → |a| →  $v = f(x)$

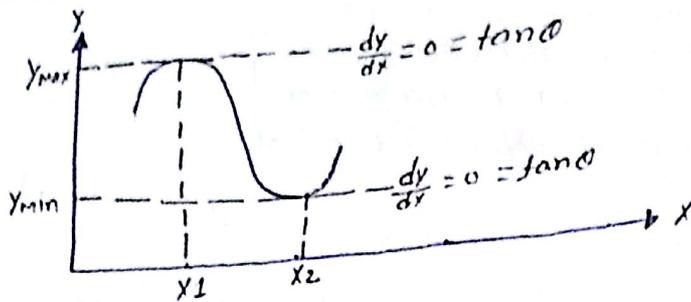
$$a = v \frac{dv}{dx} = f(x) \frac{d}{dx} [f(x)]$$

$$\begin{aligned} |b| \rightarrow v &= f(x) = \frac{dx}{dt} \\ dt &= \int \frac{dx}{f(x)} \end{aligned}$$

$$\begin{aligned} |c| \rightarrow f &= f(x) \\ v \frac{dv}{dx} &= f(x) \\ \int v dv &= \int f(x) dx \end{aligned}$$



# Maxima-minima concept



Step-I

$$\frac{dy}{dx} = 0 \Rightarrow x = x_1, x_2, x_3, \dots$$

Step-II

$$* \frac{d}{dt} \left( \frac{dy}{dx} \right) = \frac{d^2y}{dx^2} \begin{cases} x = x_1 \Rightarrow \frac{d^2y}{dx^2} \Rightarrow \ominus ve \Rightarrow \text{Max at } x_1. \\ x = x_2 \Rightarrow \frac{d^2y}{dx^2} \Rightarrow \oplus ve \Rightarrow \text{Min at } x_2. \end{cases}$$

Step-III

$$* (Y_{max}) \text{ at } x_1 \Rightarrow f(x_1)$$

$$* (Y_{min}) \text{ at } x_2 \Rightarrow f(x_2)$$

## 141 -> Equation of Motion

# 1st  $\vec{v} = \vec{u} + \vec{a}t$

# 2nd  $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

# 3rd  $\vec{v}^2 = \vec{u}^2 + 2\vec{a} \cdot \vec{s}$

# 4th  $S_{nth} = u + \frac{1}{2}a(2n-1)$   
(Dist. & Disp. in nth sec)

**NOTE** -> Equation of motion use only when particle move with const. acceleration.

$$\vec{s} = \vec{v}_{avg} \times t$$

$$\vec{v}_{avg} = \frac{\vec{s}}{t}$$

**Standard NOTE**

(T, 2T, 3T)

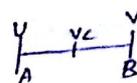
\* If particle start from rest with constant acceleration then ratio of distance covered in consecutive same time interval will be 1:3:5:7:9:...

\* If constant retardation applied on object & final velocity become zero then ratio of distance covered in a consecutive same time interval will be 9:7:5:3:1

\* If particle start from rest with constant acceleration then ratio of time taken for same consecutive distance is  $1: \sqrt{2}-1: \sqrt{3}-\sqrt{2}: \sqrt{4}-\sqrt{3}: \dots$

\* If two velocity at point A & B are given then velocity at centre of point A & B are

$$v_c = \sqrt{\frac{u^2 + v^2}{2}}$$



\* If a bus is at a distance of 'x' meter from the person to catch it, then velocity of the person to catch it from bus.

$$v \geq \sqrt{2ax}$$

\* A vehicle travel half of distance 'l' with  $v_1$  & the other half with speed  $v_2$ , then Average speed.

$$v_{avg} = \frac{2v_1v_2}{v_1+v_2}$$

\* If a particle is Accelerated for time 't<sub>1</sub>' with acceleration  $a_1$  & for a time 't<sub>2</sub>' with acceleration  $a_2$ , then Average acc<sup>n</sup>.

$$a_{avg} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$$

## MOTION UNDER GRAVITY

Case-I → Particle projected vertically upward from the ground

\* Time taken from bottom to top =  $\frac{u}{g}$

\* Time of flight ( $T_{up} + T_{down}$ )

$$T = \frac{2u}{g}$$

\*  $H_{max} = \frac{u^2}{2g}$  [Dist =  $2H_{max} = \frac{u^2}{g}$ , Disp = 0]

\* velo at time 't'

$$t = 0 \Rightarrow v = u$$

$$t = \frac{u}{g} \Rightarrow v = 0$$

$$t = \frac{2u}{g} \Rightarrow v = -u$$

\* position at time 't'

$$h = ut - \frac{1}{2}gt^2$$

\* Friction = 0  $\Rightarrow$   $t_{up} = t_{down}$   
 $T = 2t_{up} = \frac{2u}{g}$

\* Friction  $\neq 0$

$$t_{up} < t_{down}$$

$$t_{up} = \frac{u}{g+a}$$

$$t_{down} = \frac{u}{g-a}$$

\*\*\* Standard  
 NOTE → \* If air friction is not consider then, time of ascending is equal to time of descending.

\* Equation of motion in motion under gravity is only valid/use when gravitation acc<sup>n</sup> is const. It is possible when 'h' is less than 'R' ( $h \ll R$ ).

\* In last second of Ascending & First second of descending distance covered by particle is 5m & It is independent from max height & projection velocity.

\* Velocity of particle in last second of Ascending & After First second of descending is 9.8 m/sec. It is also independent from max height & projection velocity.

\* At same level speed of particle same due to direction. Velocity is changed.

\* If Air drag is consider than time of descending is more than time of Ascending.

\* Particle crosses same height at two time in journey. Its same height speed of particle is same when it goes upward & downward.

\* Distance covered by particle in last 't' sec. of Ascending & 1st 't' sec. of descending is same ( $\frac{1}{2}gt^2$ ).

Case-II → Vertical projection from Height.

$$H_{max} = H + \frac{u^2}{2g}$$

\* velocity at bottom

$$v_b = \sqrt{u^2 + 2gH}$$

\* Time of flight

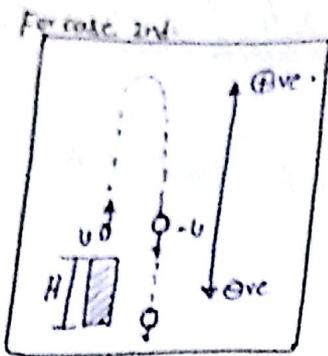
$$T = \frac{u}{g} + \sqrt{\frac{u^2}{g^2} + \frac{2H}{g}}$$

\* velo at time 't'

$$v = u - gt$$

\* position at time 't'

$$\text{Dist from Height (Building)} \rightarrow h = ut - \frac{1}{2}gt^2$$



# downward direction

\* velocity at bottom  

$$V_b = \sqrt{u^2 + 2gh}$$

\* time of flight

$$T = \frac{-u}{g} + \sqrt{\frac{u^2}{g^2} + \frac{2H}{g}}$$



Case-III → Drop from Height



\* velocity at bottom

$$V_b = \sqrt{2gh}$$

\* Time of flight

$$T = \sqrt{\frac{2H}{g}}$$

\* velocity at time 't'

$$V = -gt$$

\* position at time 't'  
 or, vertical disp. at time 't'

$$h = \frac{1}{2}gt^2$$

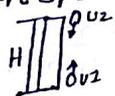
\*\*\*  
 NOTE →

- \* From same height different particle, is projected with same speed in different direction than speed of particle at bottom point independent from projection direction. ( $V = \sqrt{u^2 + 2gh}$ )
- \* If particle drop from height  $s = ut + \frac{1}{2}at^2$  ratio of dist. traveled in  $-h = (0) + \frac{1}{2}(-g)t^2$  consecutive same time interval is  $1 : 3 : 5 : 7 : \dots$
- \* If particle drop from height than ratio of time taken by particle to cover equal distance:  
 $t_1 : t_2 : t_3 = 1 : \sqrt{2} : \sqrt{3} : \sqrt{4} - \sqrt{3} : \dots$

### STANDARD RESULT

\* → When a particle is released from height with velocity  $u_2$  & another particle is projected with speed  $u_1$ . They will meet at time.

$$t = \frac{H}{u_1 + u_2}$$



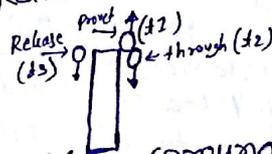
\* → When a particle is projected upward some height & another particle projected from ground than they will meet at time 't'

$$t = \frac{H}{u_2 - u_1}$$



\* → When a particle projected vertically upward from height 'h' with speed 'u' & when another particle is thrown vertically downward with same speed at same instant & at same instant another 3rd particle is released from height then relation of time taken by particle to reach the ground is.

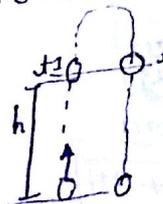
$$t_3 = \sqrt{t_1 t_2}$$



\* → Particle projected vertically upward from ground time taken the same height resp.  $t_1$  &  $t_2$ .

\*  $h = \frac{1}{2}gt_1 t_2$       \*  $u = \frac{g}{2}(t_1 + t_2)$

\*  $v = \frac{g}{2}(t_2 - t_1)$       \*  $H_{max} = \frac{g}{8}(t_1 + t_2)^2$

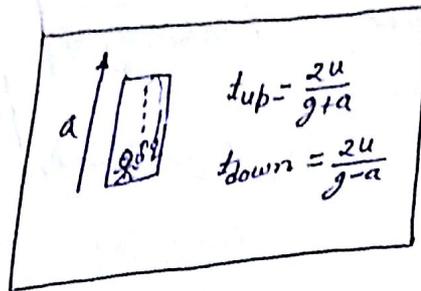


\*  $\rightarrow$  Drop in lift

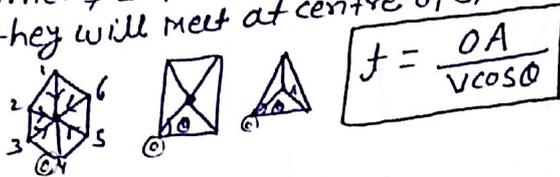
|a|  $\rightarrow$    $t = \sqrt{\frac{2H}{g+a}}$

|b|  $\rightarrow$    $t = \sqrt{\frac{2H}{g-a}}$

|c|  $\rightarrow$   $\begin{matrix} v=c \\ 0 \\ v=0 \end{matrix}$    $t = \sqrt{\frac{2H}{g}}$



\* If some particle is  $\oplus$  in square, triangle, rectangular Hexagonal corner & 1st move towards 2nd, 2nd move towards 3rd, 3rd move towards 4th like that then they will meet at centre of system like that

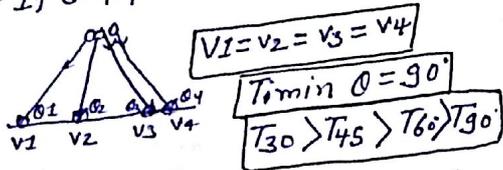


Case IV  $\rightarrow$  Motion along the Inclined Plane

\* Velocity at ground  $= \sqrt{2gH} < \theta$

\* Time taken to reach the ground  $t = \frac{1}{\sin \theta} \sqrt{\frac{2H}{g}}$

\* If drop from same height



NOTE  $\rightarrow$  \* In a translational motion along the incline plane velocity at bottom is independent from angle of inclination & size & shape of object.

\* It depend ~~only~~ at height.

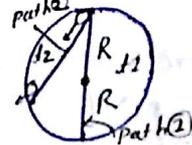
\* Time taken by particle is depend on angle of inclination & Height. but also independent from size & shape of object.

# A particle of mass 'm' move along the path 1 & 2 in vt. plane.

\* Time taken to cross the chord  $= \sqrt{\frac{2R}{g}}$

\* velo of particle at opposite chord  $= 2v \cos \theta \sqrt{Rg}$

\* Ratio of  $t_1 : t_2 = 1 : 1$

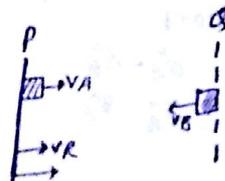


# River crossing problem

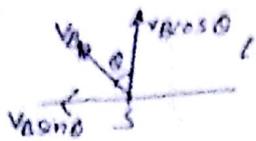
\* upstream condition  $t = \frac{PB}{v_B - v_R}$

\* For downstream condition

$t = \frac{PB}{v_B + v_R}$



\* For min distance condition



$$v_B \sin \theta = v_R \rightarrow \text{For min distance}$$

$$\theta = \sin^{-1} \frac{v_R}{v_B}$$

$$\text{Angle from River flow} = 90 + \sin^{-1} \frac{v_R}{v_B}$$

Time to cover min distance

$$t = \frac{W}{\sqrt{v_B^2 - v_R^2}}$$

\* Minimum time Approach

\* To cross river minimum time boat or swimmer move  $\perp$  to the direction of River Flow



$$\text{Drift} = x = v_R t_{\min}$$

$$= v_R \left( \frac{W}{v_B} \right)$$

$$d_{\min} = \frac{W}{v_B}$$

$$\alpha = \tan^{-1} \frac{v_B}{v_R}$$

# Rain man problem



$$v_{rg} = -y\hat{j}$$

$$v_{rm} = -y\hat{j} - x\hat{i}$$

$$|v_{rm}| = \sqrt{x^2 + y^2}$$

$$\tan \theta = x/y$$

To solve the prob. of Rain

$$\vec{v}_{rm} = \vec{v}_{rg} - \vec{v}_{mg}$$

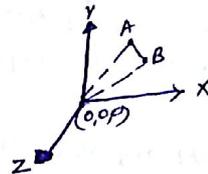
\* Ist stop the observer & then make him moving.

\* Then go on drawing the Rain on him as ray & try to make a triangle. Then find out unknown from triangle.

# Relative velocity

\* Position of 'A' w.r.t 'B'

$$\vec{x}_{AB} = \vec{x}_A - \vec{x}_B$$



\* Relative velocity

$$\text{of 'A' w.r.t 'B'} \Rightarrow \vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

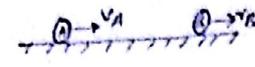
$$\text{of 'B' w.r.t 'A'} \Rightarrow \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

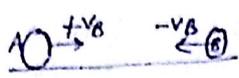
$$\vec{v}_{AB} = -\vec{v}_{BA}$$

\* Relative Acceleration

$$* \vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

$$* \vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

1a)  $\rightarrow$    $V_{AB} = v_A - v_B$   $V_{BA} = v_B - v_A$   $\left. \vphantom{\begin{matrix} V_{AB} \\ V_{BA} \end{matrix}} \right\} v_{\min}$

1b)  $\rightarrow$    $V_{AB} = v_A - (-v_B) = v_A + v_B$   $V_{BA} = -v_B - v_A = -(v_A + v_B)$   $\left. \vphantom{\begin{matrix} V_{AB} \\ V_{BA} \end{matrix}} \right\} v_{\max}$

NOTE  $\rightarrow$  \* If two particles in same direction w.r.t. common frame of reference, relative velocity is min & when move in opposite direction, relative velocity is max.  
 \* If particle move in moving frame of reference than relative velocity w.r.t. ground in same direction is max & opposite direction is min.